influence is especially large for cooled surface if its temperature decreases downstream. Consequently, the application of such a temperature distribution is more effective for laminarization compared to uniformed distribution.

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EFFECT OF SUCTION ON LAMINAR COMPRESSIVE FLOW AND HEAT TRANSFER CLOSE TO A DISK ROTATING IN A GAS

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UDC 532.526.75

Suction for gas flowing over a wall may be used to combat flow instability in the region of the leading edge of a wing [1], and the nature of flow destabilization in which there is similar development of instability in the boundary layer on a rotating disk [2]. Suctioning of the boundary layer flowing over bodies or rotating surfaces is also an effective method for intensifying heat and mass transfer processes [3]. A knowledge of hydrodynamic and thermal characteristics of the boundary layer on a rotating disk is also necessary in a whole series of other technical situations [4].

In order to study uncompressed laminar flow close to a rotating disk with different external conditions, a method has been used successfully for averaging nonlinear inertial terms in equations of motion over the thickness of a boundary layer (the Slezkin–Targ method) making it possible to obtain analytical relationships for flow characteristics required in carrying out engineering calculations [5-8]. In the current work on the basis of a modified Slezkin–Targ method a study is made of a laminar boundary layer on an infinite disk rotating in a gas with presence of uniform suction from its surface taking account of medium compressibility. A process is considered for heat exchange between the disk and the external flow. Calculations are made for the thickness of hydrodynamic and thermal boundary layers, and also values of the coefficient of the disk resistance moment c_M and Nusselt number Nu in relation to suction parameter and the ratio of temperature in the external flow and in the disk. It is demonstrated that suction markedly affects the profile of hydrodynamic flow at the disk surface, increasing its resistance moment and heat emission. Results of calculating c_M are compared with known data for accurate solution of equations for a boundary layer on a rotating disk in the case of an incompressible liquid.

1. Ignoring viscous dissipation [5, 9] equations for spatial hydrodynamic and thermal boundary layers on a rotating disk in generally accepted notations are written in the form

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right); \quad (1.1)$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}\right)=\frac{\partial}{\partial z}\left(\eta\frac{\partial v}{\partial z}\right); \quad (1.2)$$

$$\frac{\partial}{\partial r}(\rho ru)+\frac{\partial}{\partial z}(\rho rw)=0; \quad (1.3)$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 48-52, March-April, 1987. Original article submitted December 19, 1985.

$$\frac{\partial p}{\partial z} = 0; \qquad (1.4)$$

$$\rho c_p \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial z} \left(\varkappa \frac{\partial T}{\partial z} \right); \qquad (1.5)$$

$$p = \rho \frac{RT}{\mu}. \qquad (1.6)$$

Here u, v, and w are radial, azimuthal, and axial velocity components for the medium; p is pressure; ρ is density; T is temperature; c_p is specific thermal capacity at constant pressure; μ is molecular weight of the gas; R is universal gas constant; η and $\varkappa$ are coefficients of dynamic viscosity and thermal conductivity for the medium; z is axial coordinate reckoned from the disk surface.

System of equations (1.1)-(1.6) should be solved with boundary conditions

$$z = 0: u = 0,\ v = \omega r,\ w = -k,\ T = T_0,$$
$$z \to \infty: u \to 0,\ v \to 0,\ T \to T_1,$$

where ω is disk rotation angular velocity; k is suction velocity at its surface; T_0 is disk temperature; T_1 is temperature of the gas in the external flow.

Let the temperature be independent of radial coordinate [4]. As is normal, we also ignore the change in specific heat capacity c_p.

Similar to [10], we introduce a Dorodnitsyn transform

$$Z = \int_0^z \frac{\rho(z)}{\rho_1} dz \qquad (1.7)$$

(ρ_1 is gas density in the external flow).

Then the expression for transforming the axial velocity component w_1 takes the form

$$w_1 = w\rho/\rho_1 + u\partial z/\partial r. \qquad (1.8)$$

Assuming that $u = rF(Z)$, $v = rG(Z)$, $T = T_0 + (T_1 - T_0)\theta(Z)$, $\partial p/\partial r = 0$, $\eta = \eta_1 T/T_1$, $\varkappa = \varkappa_1 T/T_1$ (η_1 and $\varkappa_1$ are coefficients of dynamic viscosity and thermal conductivity in the external flow) and using (1.1)-(1.8), we obtain

$$F^2 + w_1 F' - G^2 = \nu_1 F''; \qquad (1.9)$$

$$2FG + w_1 G' = \nu_1 G''; \qquad (1.10)$$

$$2F + w_1' = 0; \qquad (1.11)$$

$$w_1 \theta' = \chi_1 \theta''. \qquad (1.12)$$

Here $\chi_1 = \varkappa_1/(\rho_1 c_p)$; $\nu_1 = \eta_1/\rho_1$; a prime indicates differentiation for Z.

In order to solve system (1.9)-(1.12) we introduce the notation $w_1 = w_0 - k_1$, in which $k_1 = \rho_0 k/\rho_1$, ρ_0 is gas density at the disk surface.

By separating in the right-hand parts of the equations terms proportional to k_1, system (1.9)-(1.12) is transformed to

$$\frac{1}{\nu_1}(F^2 + w_0 F' - G^2) = F'' + \frac{F'}{l}; \qquad (1.13)$$

$$\frac{1}{\nu_1}(2FG + w_0 G') = G'' + \frac{G}{l}; \qquad (1.14)$$

$$2F + w_0' = 0; \qquad (1.15)$$

$$\frac{1}{\chi_1} w_0 \theta' = \theta'' + \frac{\theta'}{l_1}, \qquad (1.16)$$

where $\ell = \nu_1/k_1$; $\ell_1 = \chi_1/k_1$.

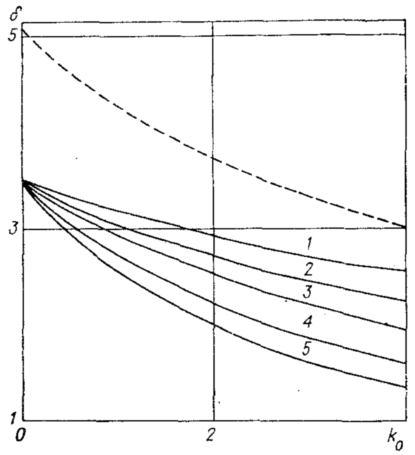

Fig. 1

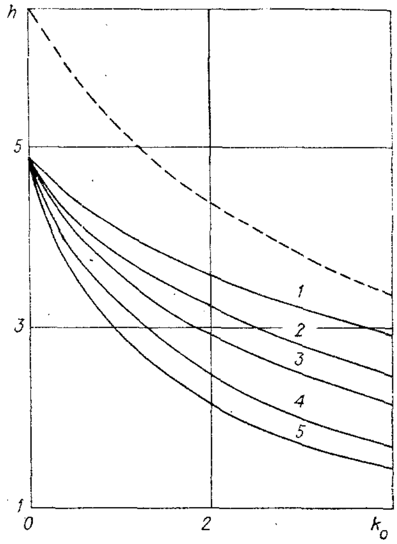

Fig. 2

By substituting terms in the left-hand parts of Eqs (1.13), (1.14) and (1.16) by their average values through the thickness of the hydrodynamic δ_1 and thermal h_1 boundary layers taking account of (1.15), we have

$$\frac{3}{\nu_1\delta_1}\int_0^{\delta_1} F^2 dZ - \frac{1}{\nu_1\delta_1}\int_0^{\delta_1} G^2 dZ = \frac{d^2F}{dZ^2} + \frac{1}{l}\frac{dF}{dZ}; \tag{1.17}$$

$$\frac{4}{\nu_1\delta_1}\int_0^{\delta_1} FG dZ = \frac{d^2G}{dZ^2} + \frac{1}{l}\frac{dG}{dZ}; \tag{1.18}$$

$$-\frac{2}{\chi_1 h_1}\int_0^{h_1}\left(\int_0^{Z} F dZ\right)\frac{d\theta}{dZ}dZ = \frac{d^2\theta}{dZ^2} + \frac{1}{l_1}\frac{d\theta}{dZ}. \tag{1.19}$$

Formally Eqs. (1.17)-(1.19) coincide with system (5) from [8], devoted to studying the case of an incompressible liquid. Therefore, for values of F, G, and θ it is possible to use expressions (7)-(9) from the work indicated, in which coordinate z is replaced by function $Z = \int_0^z \frac{\rho(z)}{\rho_1} dz$.

With $\rho = \rho_1 = \rho_0 = \text{const}$ and $k \to 0$ ($\ell \to \infty$) relationships for radial and circumferential velocity component are transformed into a Targ solution described by power polynomials [5]. In the other limiting case of considerable suction ($k \to \infty$, $\ell \to 0$, $\ell_1 \to 0$) the expressions for F, G, and θ coincide with accurate solution of boundary layer equations [8].

In order to determine unknowns A_1, δ_1, and h_1 it is necessary to solve a system of equations similar to set (10)-(12) in [8] with p = 0.

For a changeover to actual values of δ_r and h_r it is necessary to use the relationships

$$\frac{\delta_r\left(\frac{\omega}{\nu_1}\right)^{1/2}}{\delta_0} = \frac{1}{n} + \frac{1-\frac{1}{n}}{[\exp(-f)+f\exp(-f)-1]}\left[\frac{d\,\mathrm{Pr}}{2}\exp(-f) - \frac{\exp(-d\,\mathrm{Pr})}{d\,\mathrm{Pr}} - 1 + \frac{1}{d\,\mathrm{Pr}}\right]; \tag{1.20}$$

$$\frac{h_r\left(\frac{\omega}{\nu_1}\right)^{1/2}\mathrm{Pr}}{h_0} = \frac{1}{n} + \frac{1-\frac{1}{n}}{[\exp(-f)+f\exp(-f)-1]}\left[\frac{f}{2}\exp(-f) - \frac{\exp(-f)}{f} - 1 + \frac{1}{f}\right]; \tag{1.21}$$

where $\delta_0 = \delta_1(\omega/\nu_1)^{1/2}$; $h_0 = h_1(\omega/\nu_1)^{1/2}\mathrm{Pr}$; $A_0 = A_1(\nu_1/\omega^2)$; $d = k_0\delta_0 n$; $f = k_0 h_0 n$; $n = T_1/T_0$; $k_0 = k(1/\omega\nu_1)^{1/2}$; $\mathrm{Pr} = \nu_1/\chi_1$ is Prandtl number.

Calculated dependences for values of δ_0 and h_0 on suction parameter k_0 for $n = T_1/T_0 =$ 0.5; 0.75; 1; 1.5; 2 (lines 1-5) and Pr = 1 are presented in Figs. 1 and 2. As an example

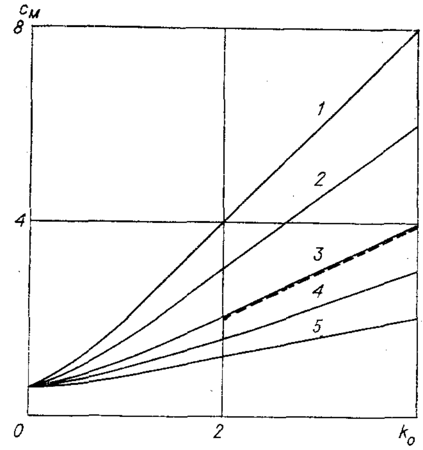

Fig. 3

TABLE 1

n	Nu					
	k_0					
	0.1	0.5	1.0	2.0	3.0	4.0
0.5	0.436	0.550	0.717	1.112	1.560	2.033
0.75	0.449	0.644	0.906	1.560	2.275	3.012
1.0	0.463	0.717	1.112	2.033	3.012	4.006
1.5	0.491	0.906	1.560	3.012	4.504	6.002
2.0	0.520	1.112	2.033	4.006	6.002	8.001

similar dependences are shown by broken lines for dimensionless actual thicknesses of hydrodynamic and thermal boundary layers found in accordance with (1.20) and (1.21) with n = 0.5.

Suction causes an additional inflow from the outer region towards the disk of gas with a rotation velocity and temperature differing from values which the medium has close to the surface. This leads to a reduction in thickness of hydrodynamic and thermal boundary layers with an increase in parameter k_0 characterizing suction intensity. An increase in the ratio of temperature in the external flow T_1 and in the disk T_0 ($n = T_1/T_0$) with uniform stable conditions promotes an increase in the efficiency of suction as a result of a relative increase in suctioned gas density, which also leads to a drop in boundary layer thickness.

It is noted that the approximate method used for calculation is based on averaging the left-hand parts of Eqs. (1.13), (1.14), and (1.16) through the thickness of the hydrodynamic and thermal boundary layers. For this reason its accuracy with an increase in k_0 increases due to a relative reduction in the role of average terms [8]. By using the relationship (1.20) found, (7) from [8], and continuity equation (1.3), it is possible to obtain an analytical expression connecting axial flow at a distance from the disk surface and parameter n.

2. By using an expression for azimuthal velocity component, we calculate the moment of frictional force M_0 operating on one side of the surface of a disk of radius R_0:

$$M_0 = 2\pi\eta_1 \int\limits_0^{R_0} r^3 \left|\frac{\partial G}{\partial z}\right|_{z=0} dr = \frac{\pi\eta_1\omega R_0^4 (\omega/\nu_1)^{1/2}\, dn\,[\exp(-d)-1]}{2\delta_0\,[1-(1+d)\exp(-d)]}.$$

Given in Fig. 3 are dependences for the coefficient of friction moment $c_M = 2M_0/\pi\eta_1R_0{}^4(\omega^3/\nu_1)^{1/2}$ on k_0 with n = 2; 1.5; 1; 0.75; 0.5 (lines 1-5), and shown by a broken line are the results of calculating c_M obtained on the basis of accurate solution of equations for the boundary layer with large k_0 and n = 1 [4]. It can be seen that suction sharply increases disk resistance, and with high values of k_0 the relationship $c_M \simeq k_0n$ is valid. An increase in coefficient for friction moment with an increase in parameter n in the case of $k_0 \neq 0$ is explained by an increase in the efficiency of suction as a result of an increase in suctioned gas density. In the other limiting case ($k_0 \to 0$) the value of c_M is independent of n. This is connected with the situation that with disk cooling a reduction in dynamic viscosity close to its surface is compensated by an increase in the axial gradient of azimuthal velocity caused by a drop in actual boundary layer thickness. The calculated value of c_M equals 0.579 with $k_0 = 0$ almost coincides with the results in [10].

TABLE 2

Pr	h_0/Pr	Nu	Nu*	Pr	h_0/Pr	Nu	Nu*
1.0	4.861	0.415	0.400	0.85	5.389	0.374	0.365
0.95	5.010	0.402	0.389	0.8	5.649	0.357	0.353
0.9	5.183	0.389	0.377	0.75	5.951	0.341	0.340

3. Now we estimate local Nusselt number characterizing the intensity of heat flow $q = -\varkappa \partial T/\partial z$ in the disk surface. By using expression (9) from [8], we find that

$$\mathrm{Nu} = \frac{q(0)}{(T_0 - T_1)\varkappa_1}\left(\frac{\nu_1}{\omega}\right)^{1/2} = \frac{\Pr f\,[\exp(-f) - 1]}{h_0\,[\exp(-f) + f\exp(-f) - 1]}.$$

Results of calculating Nu for different values of k_0 and n with Pr = 1 are given in Table 1. As follows from the dependences obtained, suction promotes an increase in disk heat transfer. With large suction parameters the value of Nu emerges into an asymptote $\mathrm{Nu} = \Pr k_0 n$.

Presented in Table 2 are data for calculating the dimensionless thickness of thermal boundary layer h_0/Pr and Nu for different values of Pr with $k_0 = 0$ and $n = 1$. Also given here for comparison are the results of calculating Nu* from [11].

In conclusion we note the simplicity and clarity of the approach used in the present work to analyzing boundary layer equations, making it possible to obtain analytical expressions for hydrodynamic and thermal characteristics of compressed flow, and also velocity and temperature profiles over the whole range of change in parameter k_0.

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